Asymptotic Behavior of Discretely Attached Corrugated Shear Webs

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Corrugated shear webs are considered that are many corrugations wide, obey Hooke's law, and have discrete (rather than continuous) attachments to framing members (e.g., spar caps) at the ends of the corrugations. A simple deformation model is postulated for such webs, based on the assumption that they differ from continuously attached webs mainly by a St. Venant effect. Analysis of this model leads to simple formulas for the shear stiffness and maximum transverse flexural strain as functions of the corrugation length. These formulas are compared with the results of more accurate analyses and found to be in excellent agreement with them, even for relatively small length-to-pitch ratios. It is suggested that those formulas can lead to considerable economies in theoretical and experimental studies of corrugated plates in shear, for they will permit one to infer data for a large range of length-to-pitch ratios from calculations or tests on just a few length-to-pitch ratios.

Nomenclature

L	= length of corrugations
p	= pitch of corrugations
p'	= developed width of one corrugation
t	= thickness of sheet from which corrugations are formed
G	= shear modulus of material
F	= resultant shear force on cross sections parallel to corrugations
$\gamma = F/LtG$	= uniform shear strain in material in case of continuous attachment
$\bar{\gamma}_I = \gamma p'/p$	= apparent shear strain of continuously attached web due to F
$\bar{\gamma}$	= apparent shear strain of discretely attached web due to F
$G' = F/L\bar{\gamma}$	= shear stiffness definition for discretely attached web
$G_{\text{eff}} = F/Lt\bar{\gamma}$	= alternate shear stiffness definition for discretely attached web
$G'_I = F/L\bar{\gamma}_I$	= shear stiffness corresponding to G' in case of continuous attachment
$\Omega = \bar{\gamma}_I/\bar{\gamma} = G'/G'_I$	= ratio of shear stiffness of discretely attached web to that of same web continuously attached
$\epsilon_{t_{\max}}$	= maximum extreme-fiber transverse flexural strain
$\sigma_{t_{\max}}$	= maximum extreme-fiber transverse flexural stress
K_1, K_2, K_3, etc.	= constants depending on cross-sectional geometry and nature of attachments (but not upon L) $(K_5 = K_4(t/p)^{1/2})$
E	= Young's modulus of material
ν	= Poisson's ratio of material
h	= height of corrugations
f	= crest width of trapezoidal corrugation
$2e$	= trough width of trapezoidal corrugation

Introduction

CORRUGATED shear webs are considered that are many corrugations wide, have discrete (rather than continuous) attachments to the framing members at the ends of the corrugations, and obey Hooke's law. The corrugations are assumed to be sufficiently long (compared to their pitch and to the spacing of the attachments) so that the deformations of a discretely attached web, under a given shear load, will differ from those of the same web continuously attached mainly by a St. Venant effect. This permits a simple deformation model to be postulated for the discretely attached case, and the analysis of this model leads to a simple formula for the shear stiffness of the web as a function of the length of the corrugations. Another simple formula is obtained for the maximum transverse flexural strain, occurring at the ends of the corrugations, as a function of the length of the corrugations.

The simple formulas referred to above are compared with the results of accurate calculations and found to be in very good agreement with them, even for relatively small length-to-pitch ratios. It is suggested that the main use of the present work will be for extrapolation purposes; that is, to infer data for a large range of length-to-pitch ratios from calculations or experiments on just a few length-to-pitch ratios.

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Analysis and Results

Deformation Model

Figure 1a shows a repeating unit of the unloaded corrugated shear web, and Fig. 1b shows the deformation of this unit under a shear load F, assuming that the ends of the corrugations are continuously attached to semirigid diaphragms, that is, diaphragms which preserve the shapes of the end cross sections but offer no restraint to the warping of these cross sections out of their planar condition. For such attachment a uniform shear strain of

$$\gamma = F/LtG \tag{1}$$

is produced in the material, giving rise to an apparent shear strain of

$$\bar{\gamma}_I = \gamma p'/p = (F/LtG)(p'/p) \tag{2}$$

for the web as a whole. In terms of the shearing displacement U_0 shown in Fig. 1b, we also have

$$\bar{\gamma}_I = U_0/L \tag{3}$$

Figure 1c shows the assumed deformation of the repeating unit under the same resultant shear force F, but with the continuous attachment replaced by discrete attachments. For

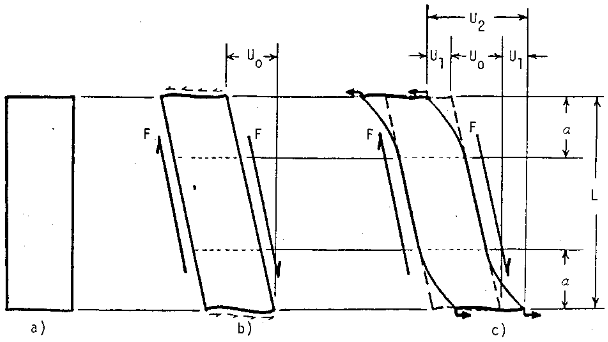

Fig. 1 Deformation model: a) Repeating unit of unloaded shear web; b) Repeating unit of loaded shear web with continuous attachment; c) Repeating unit of loaded shear web with continuous attachment replaced by discrete attachments.

comparison, the deformation in the case of continuous attachment is also shown in Fig. 1c; it is represented by the dashed lines.

As is evident from Fig. 1c, we are assuming that there is a central region, of length L-$2a$, in which the deformations of the discretely attached web and the continuously attached web are identical for the same shear load F. In this region the material shear strain and the apparent web shear strain are still as given by Eqs. (1) and (2). The differences between the deformations of the two webs are, assumedly, confined to end regions, each of length a and each giving rise to an additional shearing displacement of an amount U_1. It is further assumed that, while the length a may depend on the shape of the cross section and the nature of the attachments, it is independent of L. This assumption is based on St. Venant's principle.

Thus, in the case of discrete attachments, the apparent shear strain of the web as a whole is

$$\bar{\gamma} = U_2/L = (U_0 + 2U_1)/L \tag{4}$$

or, in view of Eq. (3)

$$\bar{\gamma} = \bar{\gamma}_1 + (2U_1/L) \tag{5}$$

Now, in structures which obey Hooke's law and have a linear relationship between load and displacements, the strains or deformations at different places maintain constant ratios to each other as the load is increased. Thus, referring to Fig. 1c, the apparent shear strain increment U_1/a occurring within each end region is proportional to the apparent shear strain $\bar{\gamma}_1$ of the central region, or

$$U_1/a = K_1\bar{\gamma}_1 \tag{6}$$

Using this to eliminate U_1 in Eq. (5) and normalizing L by dividing it by p (and other characteristic lateral dimension may be used instead), we arrive at the following expression for the apparent shear strain of the discretely attached web

$$\bar{\gamma} = \bar{\gamma}_1 \left[1 + \frac{2a}{p} \frac{K_1}{(L/p)} \right]$$

or, letting $2a/p$ be absorbed into K_1

$$\bar{\gamma} = \bar{\gamma}_1 \left[1 + \frac{K_2}{(L/p)} \right] \tag{7}$$

Shear Stiffness

The shear stiffness of the discretely attached web can be expressed in many ways. One way is through the ratio of the shear force F to the apparent shear strain $\bar{\gamma}$. Taking F from Eq. (2) and $\bar{\gamma}$ from Eq. (7), we arrive at the following formula for this stiffness

$$\frac{F}{\bar{\gamma}} = GLt \frac{p}{p'} \left[1 + \frac{K_2}{(L/p)} \right]^{-1} \tag{8}$$

Other stiffness measures that have been used in the literature on corrugated plates in shear are G' and G_{eff}, defined as follows

$$G' = (F/L)/\bar{\gamma} \qquad G_{\text{eff}} = (F/Lt)/\bar{\gamma} \tag{9}$$

Equation (8) gives the following formulas for these two stiffnesses according to the present model

$$G' = Gt \frac{p}{p'} \left[1 + \frac{K_2}{(L/p)} \right]^{-1} \tag{10a}$$

$$G_{\text{eff}} = G \frac{p}{p'} \left[1 + \frac{K_2}{(L/p)} \right]^{-1} \tag{10b}$$

A dimensionless measure of shear stiffness is the ratio Ω of the shear stiffness of the discretely attached web to that of the same web continuously attached. This is the same as the inverse of the ratio of the two flexibilities. Thus, $\Omega = \bar{\gamma}_1/\bar{\gamma}$ or, from Eq. (7),

$$\Omega = \left[1 + \frac{K_2}{(L/p)} \right]^{-1} \tag{11}$$

This dimensionless shear stiffness can be converted to the various dimensional ones through the following relationships:

$$F/\bar{\gamma} = GLt(p/p')\Omega \tag{12a}$$

$$G' = Gt(p/p')\Omega \tag{12b}$$

$$G_{\text{eff}} = G(p/p')\Omega \tag{12c}$$

Equations (8), (10), and (11) show how, according to the present model, the stiffness of a discretely attached corrugated shear web depends on the length-to-pitch ratio L/p of the corrugations. The constant K_2 in these formulas depends on the nature of the attachments and the geometry of the cross section (for example, on t/p and on the height-to-pitch ratio), but not on L/p.

It has been found advantageous in many cases to regard Ω as a function of $(L/p)(t/p)^{1.5}$, rather than as a function of L/p, for then a relationship is obtained which is relatively insensitive to t/p (see Refs. 2 to 8). To take advantage of this simplification in the present work we need only rewrite Eq. (11) as follows

$$\Omega = \left[1 + \frac{K_3}{(L/p)(t/p)^{1.5}} \right]^{-1} \tag{13}$$

where K_3 is a new constant which we would expect to be much less sensitive to t/p than is K_2. The same change can be made in Eqs. (8) and (10). (For the case of a corrugated shear web with trough lines constrained to remain straight, as though by means of attachments to a flat plate along their entire length, the findings in Ref. 1 suggest that the exponent 1.5 should be changed to 1.0)

Flexural Strain

The shearing of corrugated plates with discrete attachments gives rise to considerable transverse flexure near the ends of the corrugations, due to the deformations of the cross sections in their own planes, and the maximum value $\epsilon_{t_{\max}}$ of the extreme-fiber strain due to this flexure may be a quantity of practical interest. Assuming that the location of $\epsilon_{t_{\max}}$ does not change as L/p changes, and again invoking the reasoning that led to Eq. (6), we may write

$$\epsilon_{t_{\max}} = K_4 \bar{\gamma}_1 \tag{14}$$

This, together with Eq. (7), leads to the following form of the relationship between $\epsilon_{t_{\max}}$ and L/p

$$\epsilon_{t_{\max}} = \bar{\gamma} K_4 \left[1 + \frac{K_2}{(L/p)} \right]^{-1} \tag{15}$$

This can be rewritten in various ways. The findings in Refs. 2-7 suggest that the following form may be advantageous in that the constants appearing in it will be relatively insensitive to t/p:

$$(t/p)^{1/2} \frac{\epsilon_{t_{\max}}}{\bar{\gamma}} = K_5 \left[1 + \frac{K_3}{(L/p)(t/p)^{1.5}} \right]^{-1} \tag{16}$$

From this equation and Eq. (13) we also have

$$(t/p)^{1/2} \frac{\epsilon_{t_{\max}}}{\bar{\gamma}} \frac{1}{\Omega} = K_5 = K_4 (t/p)^{1/2} \tag{17}$$

The substitutions

$$\epsilon_{t_{\max}} \approx \sigma_{t_{\max}} (1-\nu^2)/E = \sigma_{t_{\max}} (1-\nu)/2G \tag{18}$$

$$\Omega\bar{\gamma} = (F/GLt)(p'/p) \quad \text{[from Eq. (12a)]} \tag{19}$$

convert Eq. (17) to the following more meaningful form:

$$(t/p)^{1/2} \frac{\epsilon_{t_{\max}}}{\bar{\gamma}\Omega} = \tfrac{1}{2}(1-\nu)\,(t/p)^{1/2} \frac{p}{p'} \frac{\sigma_{t_{\max}}}{(F/Lt)} = K_5 \tag{20}$$

This suggests that the ratio of the maximum extreme-fiber transverse flexural stress $\sigma_{t_{\max}}$ to the average applied shear stress F/Lt on longitudinal sections is virtually independent of corrugation length.

Verification

Equation (11) can be rewritten as

$$(1-\Omega)\ (L/p) = K_2\Omega \tag{21}$$

Thus, the present model predicts a linear relationship between $(1-\Omega)(L/p)$ and Ω for sufficiently large L/p. Figures 2 and 3 show that this prediction is in good agreement with the results of accurate analysis. The solid curves in the upper part of each figure give Ω as a function of L/p for a particular cross section and several different kinds of end attachments, according to the data of Refs. 7 and 8. The solid curves in the lower part of each figure give the same data replotted in terms of the coordinates $(1-\Omega)(L/p)$ and Ω. It is seen that in these coordinates the actual data (shown solid) and their straight-line approximations by Eq. (21) (shown dashed) are practically indistinguishable from each other once L/p exceeds a certain value, which is not necessarily very large. The approximations are also replotted in the upper part of each figure, using Eq. (11), for the sake of direct comparison with the data as originally given in Refs. 7 and 8.

A verification of Eq. (15) in the case of curvilinear corrugations is shown in Fig. 4, which pertains to the A and B cases of Fig. 2. Equation (15) is seen to be a very good approximation of the actual data. Figure 5 gives a similar verification of Eq. (15) for the trapezoidal cases that were considered in Fig. 3.

Equation (17) or (20) predicts that $\epsilon_{t_{\max}}/\bar{\gamma}\Omega$ will show much less variation with respect to L/p than does $\epsilon_{t_{\max}}/\bar{\gamma}$ or Ω alone. This prediction is borne out by Figs. 6 and 7.

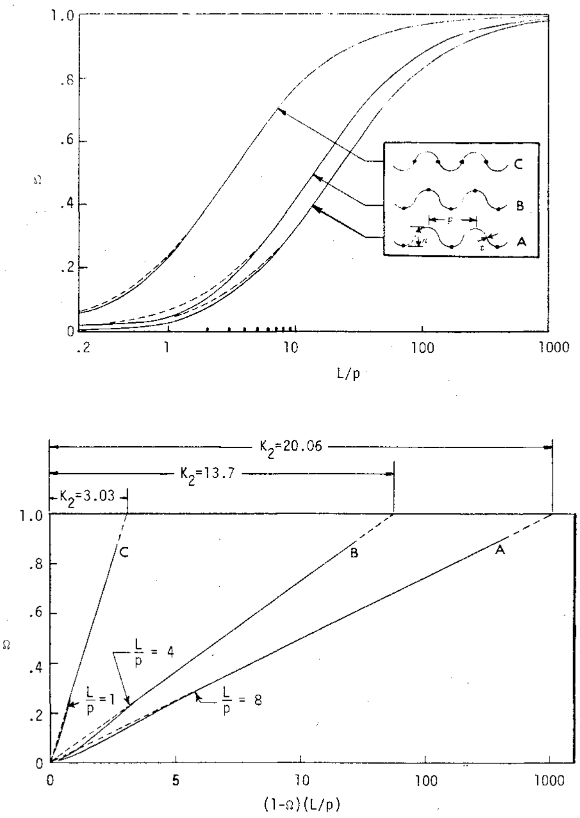

Fig. 2 Stiffness of discretely attached corrugated shear webs with circular-arc corrugations ($t/p = .015$, $h/p = 0.3$) *Solid curves:* Data from Ref. 7. *Dashed curves:* Approximation of data by Eq. (11) or (21)

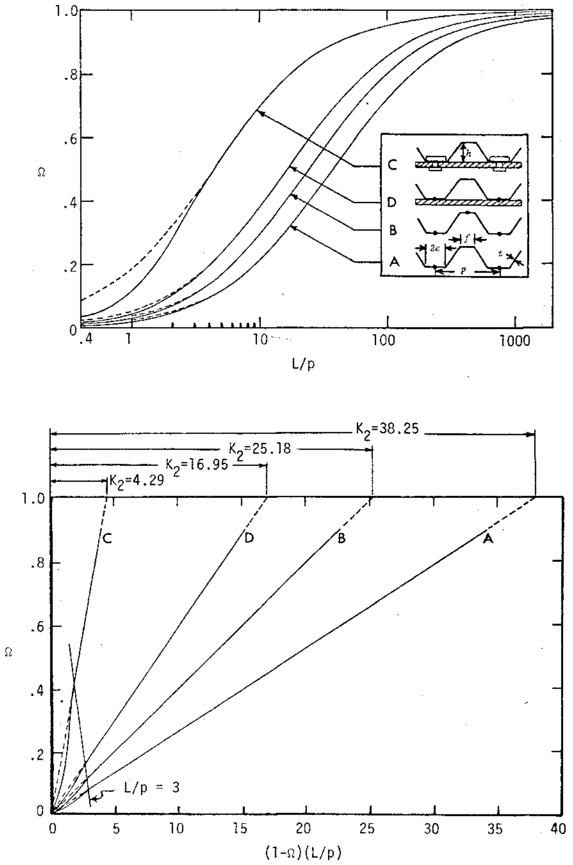

Fig. 3 Stiffness of discretely attached trapezoidally corrugated shear webs ($t/p = .015$, $h/p = f/p = 2e/p = 0.3$) *Solid curves:* Data from Ref. 8. *Dashed curves:* Approximation of data by Eq. (11) or (21).

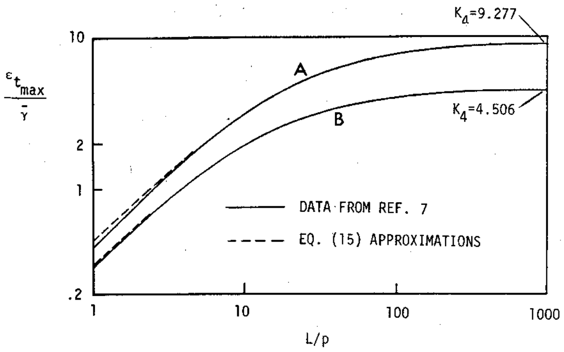

Fig. 4 Data on maximum extreme-fiber transverse flexural strain for the *A* and *B* curvilinearly corrugated shear webs of Fig. 2.

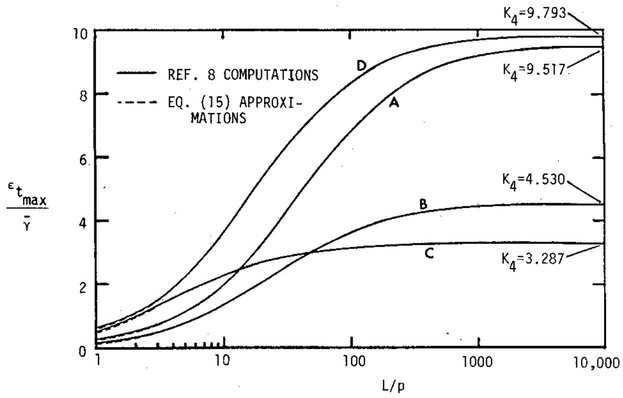

Fig. 5 Data on maximum extreme-fiber transverse flexural strain for the trapezoidally corrugated shear webs of Fig. 3.

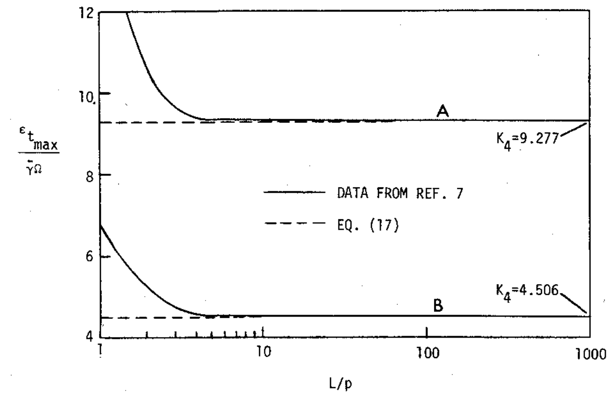

Fig. 6 Verification of Eqs. (17) and (20) for the *A* and *B* curvilinearly corrugated shear webs of Fig. 2.

Possible Applications

Theoretical Studies

The simple asymptotic formulas developed in this paper should be useful for extrapolation purposes in both theoretical and experimental studies of discretely attached corrugated shear webs. In theoretical studies any calculations pertaining to a given cross section and type of attachment may be confined mainly to the low L/p range, going only as far into the higher L/p's as is necessary to establish the constants in the asymptotic formulas. This should be especially useful to those who are using the finite-difference or finite-element method as the basis of their analyses, for in those methods the number of nodal points, and therefore the computation costs, can become very high as L/p increases.

Experimental Studies

A similar economy is possible in experimental studies of discretely attached corrugated shear webs. These, too, may

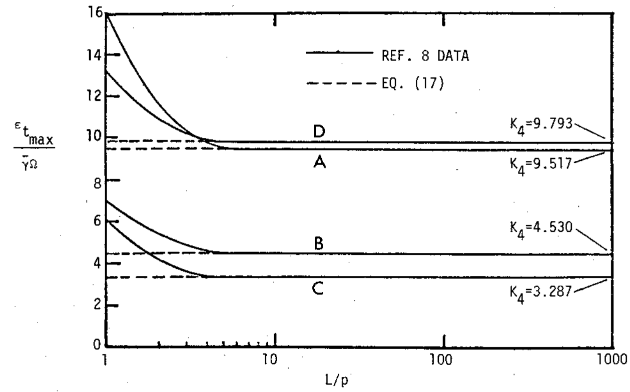

Fig. 7 Verification of Eqs. (17) and (20) for the trapezoidally corrugated shear webs of Fig. 3.

start with the small L/p's and go high enough in L/p only to establish the values of the constants in the asymptotic formulas. In experimental studies, however, there are two complications not present in theoretical ones, which will now be discussed briefly.

The first complication is that an experimental specimen may consist of two or more panels fastened together by means of side-lap fasteners along lines parallel to the corrugations, and part of the apparent shear strain $\bar{\gamma}$ may be due to deformations of those fasteners. There may be a similar contribution to $\bar{\gamma}$ from those fasteners which join the sides of the corrugated shear web to the framing members. In that case, when determining an experimental Ω by means of the formula

$$\Omega = \bar{\gamma}_I / \bar{\gamma} = G'/G'_I \tag{22}$$

one must be sure that the side and side-lap fastener deformations are also included in $\bar{\gamma}_I$, the apparent shear strain of the same web with continuous attachment at the ends of the corrugations. This can be done theoretically by using a reduced G in place of the actual G in Eqs. (1) and (2), provided that one can quantify the side and side-lap fastener deformations due to a given F. It is more than likely, however, that such a quantification will be very difficult. In that case, the following expedient is proposed: Rewrite Eq. (21) as

$$[1-(G'/G'_I)]L/p = K_2\, G'/G'_I \tag{23}$$

and regard G'_I as another curve-fitting constant to be determined, like K_2, from the experiments. That is, through several experiments measure G' as a function of L (keeping the ratios of side and side-lap fastener spacing to p constant as L is increased), then try different values of G'_I (greater, of course, than the highest measured G') until one is found that best makes the graph of

$$[1-(G'/G'_I)]L/p \quad \text{vs} \quad G'/G'_I$$

behave, asymptotically, like Eq. (23). The G'_I and K_2 thus determined will automatically include the effects of the fastener deformations. With G'_I and K_2 determined, Eq. (23) can be rewritten as

$$G' = G'_I[1+(K_2/(L/p))]^{-1} \tag{24}$$

and Eq. (24) used to predict the G' values for L/p's greater than those covered in the experiments.

The second complication in experimental studies is that part of any measured deflection will be due to axial deformations of the framing members, and this part should be subtracted to determine the deflection due to deformation of the web (and its fasteners) alone. An approximate procedure for doing this is described in the "Illustrative Application" section of Ref. 7. In applying this approximate procedure, one should include some effective width of the corrugated sheet in the cross-sectional areas of those framing members which are parallel to the corrugations—that is, in the stiff direction of the sheet. The complications due to axial deformations of the framing members can, of course, be minimized by making the cross-sectional areas of the framing members in any experiment large compared with the cross-sectional area of the web.

Conclusions

Simple formulas have been developed to show how the stiffness and flexural strains of corrugated shear webs with discrete attachments to framing members at the ends of the corrugations vary with the length-to-pitch ratio of the corrugations as this ratio increases. The validity of these formulas has been verified by comparisons of their predictions with the results of more refined analyses. The possible usefulness of the asymptotic formulas in theoretical and experimental studies of corrugated shear webs has also been discussed.

It should be noted that the asymptotic formulas for stiffness developed in the present paper are different from a corresponding formula proposed by Luttrell. Luttrell's formulation, as cited by Yu (Ref. 9, p. 384), would in essence put $(L/p)^2$ wherever the present formulas have L/p, and $(L/p)^2(t/p)^2$ in Eq. (13) where $(L/p)(t/p)^{1.5}$ now appears.

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